Glossary of Mathematical Representations

Bar Model (single) for additive reasoning

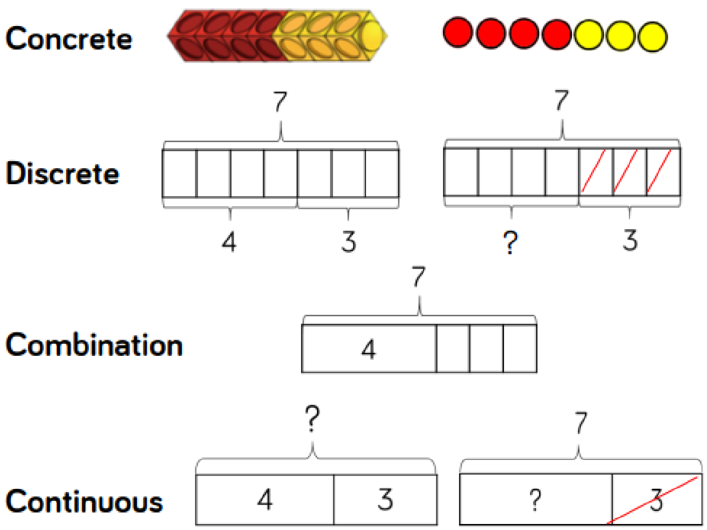
The single bar model is a type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a **concrete** representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The **combination** bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

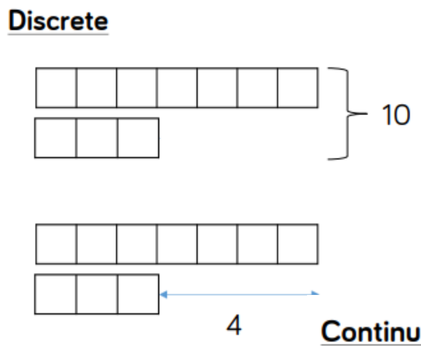


Bar Model (multiple) for additive reasoning

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right-hand side of the bars.

Smaller numbers can be represented with a **discrete** bar model whilst **continuous** bar models are more effective for larger numbers.

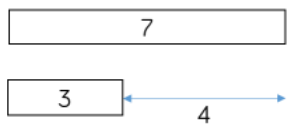


$7 + 3 = 10$

$7 - 3 = 4$

Continuous

Multiple bar models can also be used to represent the **difference** in subtraction. An arrow can be used to model the difference. When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.



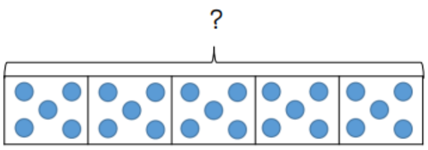
$7 - 3 = 4$

Bar Model for multiplicative reasoning

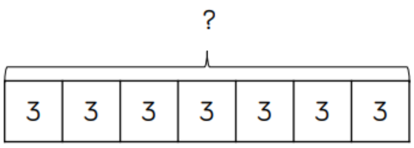
Children can use the single bar model to represent multiplication as **repeated addition**.

They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into **equal groups**. It is important when solving word problems that the bar model represents the problem.

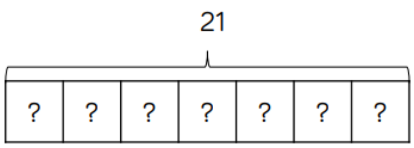


$5 \times 5 = 25$



$3 \times 7 = 21$

$7 \times 3 = 21$

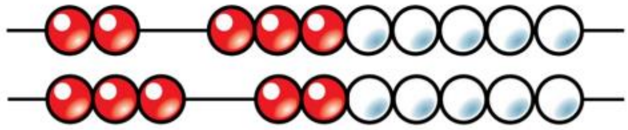


$21 \div 7 = 3$

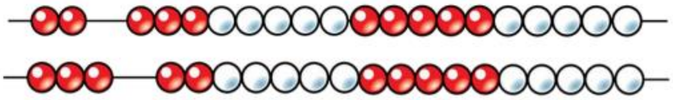
Bead strings for additive reasoning

Different sizes of bead strings can support children at different stages of addition and subtraction.

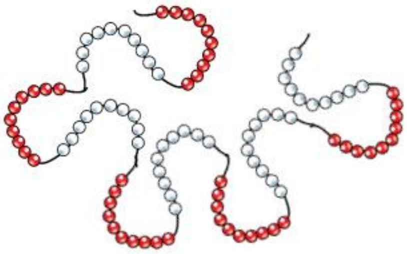
Bead strings to 10 are very effective at helping children to investigate **number bonds** up to 10. They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have **partitioned** the 10 beads into e.g. 2 + 8 = 10, move one bead, 3 + 7 = 10.



Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.



Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.



Bead strings for multiplicative reasoning

Bead strings to 100 can support children in their understanding of multiplication as **repeated addition**. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently. Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$

Children can also use the bead string to count forwards and backwards in **multiples**, moving the beads as they count.

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

When dividing, children build the number they are dividing and then **group** the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

Counting wand

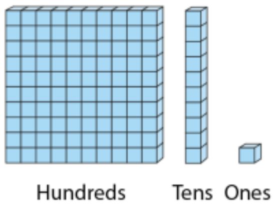
Blank counting wands support **cardinality** and counting with **1 to 1 correspondence**. Children can make their own wands and use them to count individual objects, tagging each object with a number as they count. It is important that they understand that the last number they say, tells them how many are in the group.

Numbered counting wands enable children to link numerals and amounts. For example, children can thread the correct number of beads onto the wand.

In KS1, counting wands can be used for **skip counting** and to support understanding of **unitising**.

Dienes

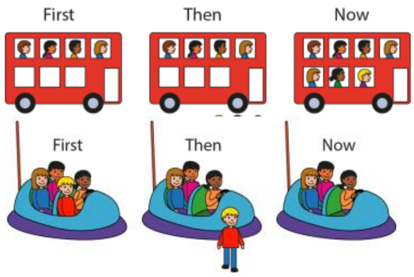
Dienes are physical blocks made of plastic or wood that represent numbers in **base ten**. They help students visualize numbers, understand place value, and perform operations like addition and subtraction in a concrete way.



First… Then… Now… stories

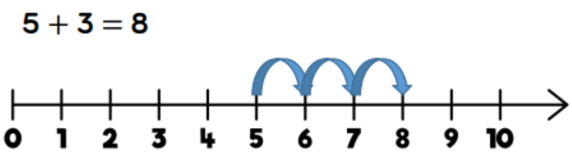
First, Then, Now stories are a mathematical tool to create a calculation story to help young children understand numbers in a meaningful context.

They use a narrative structure to represent the change in a group of objects over time: **First** describes the initial quantity, **Then** details an action (like adding more or taking some away), and **Now** shows the resulting final quantity.



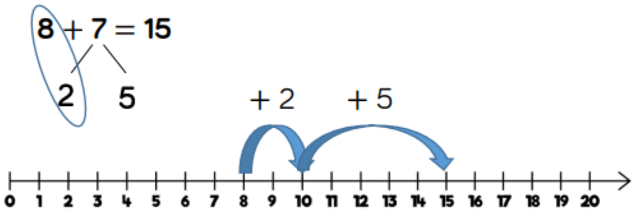
Number line for additive reasoning

Number lines support children in their understanding of addition and subtraction as **augmentation** and **reduction**.

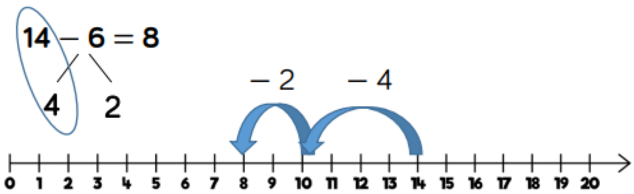


Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the **number track**.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by **ten frames**. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

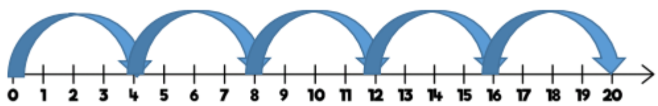


Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they **partition** the smaller number into the two separate jumps.

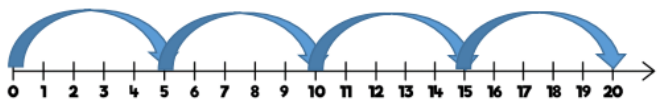


Number line for multiplicative reasoning

Labelled number lines are useful to support children to count in **multiples**, forwards and backwards as well as calculating single-digit multiplications.



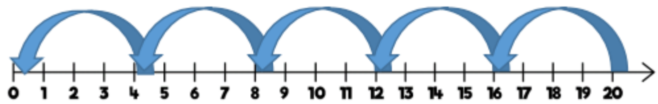
When multiplying, children start at 0 and then count on to find the product of the numbers. When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.



$4 \times 5 = 20$

$5 \times 4 = 20$

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.



$20 \div 4 = 5$

Number track for additive reasoning

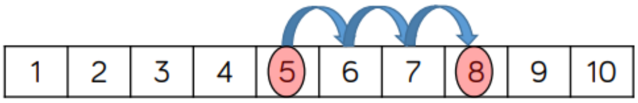
Number tracks are useful to support children in their understanding of **augmentation** and **reduction**.
When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the **minuend** and then take away the **subtrahend** to find the difference between the numbers.

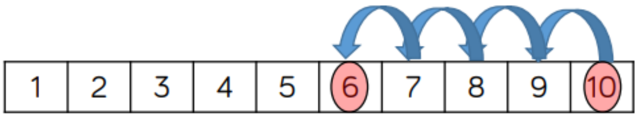
Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

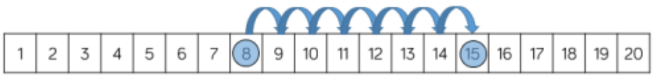
$5 + 3 = 8$



$10 - 4 = 6$



$8 + 7 = 15$

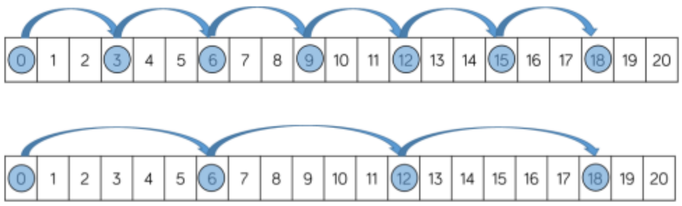


Number track for multiplicative reasoning

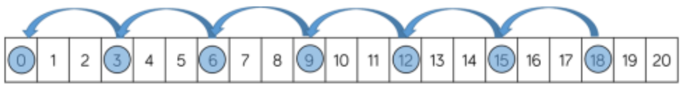
Number tracks are useful to support children to count in **multiples**, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers. When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.



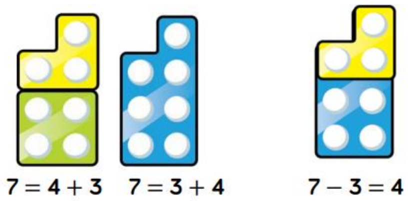
$6 \times 3 = 18$
$3 \times 6 = 18$



$18 \div 3 = 6$

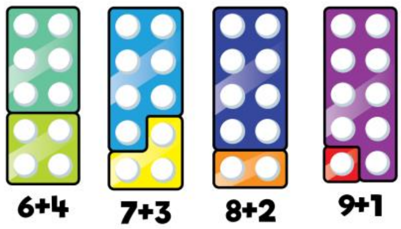
Numicon/Number Shapes for additive reasoning

Number shapes can be useful to support children to **subitise** numbers as well as explore **aggregation**, **partitioning** and **number bonds**.



When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.



Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

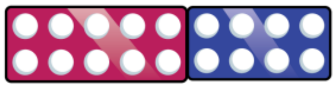
Numicon/Number Shapes for multiplicative reasoning

Number shapes support children's understanding of multiplication as **repeated addition**.

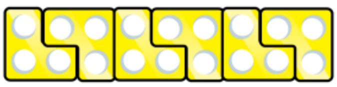
$5 \times 4 = 20$
$4 \times 5 = 20$

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd × odd = even, odd × even = odd, even × even = even.

$5 \times 4 = 20$
$4 \times 5 = 20$



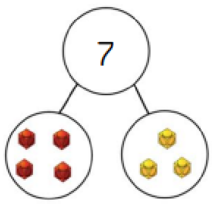
$18 \div 3 = 6$

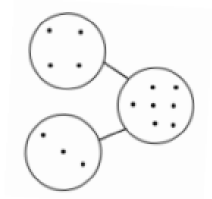


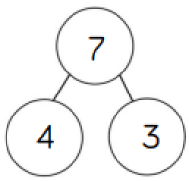
When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18

Part Whole (Cherry) Model

This part-whole model supports children in their understanding of **aggregation** and **partitioning**. Due to its shape, it can be referred to as a cherry part-whole model.





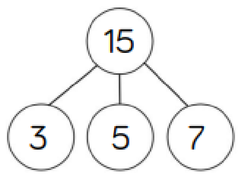


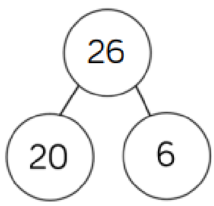
$7 = 4 + 3$
$7 = 3 + 4$

$7 - 3 = 4$
$7 - 4 = 3$

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

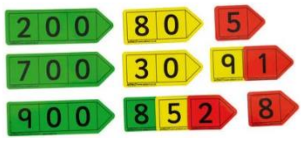
When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.





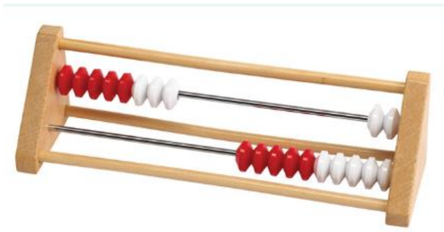
Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

Place Value Counters and Cards

Place value counters and cards represent numbers (e.g. 1, 10, 100) and are positioned to help children learn and visualize the concept of place value in mathematics.

Rekenrek



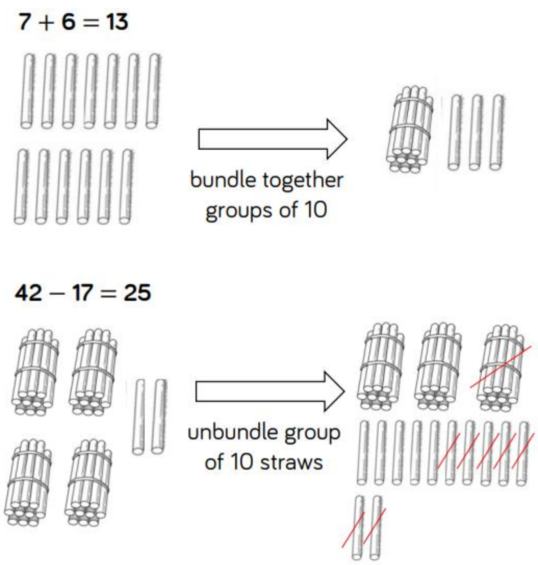
A rekenrek is a counting frame used in early maths to help children build a sense of number. A rekenrek is typically made up of two rows of rows of 10 beads; the first five are red and the second five are white.

The primary aim of the rekenrek is not to calculate the answer to a question, but to allow the children to **see** the maths and understand what happens as an answer is being calculated. The parallel bars of beads allow children to compare numbers. Rekenreks also help children to understand how whole numbers can be made up of 2 parts.

The rekenrek helps children learn essential maths principals, such as **counting**, **subitising**, **place value**, **number bonds** and **additive reasoning**. It can be used to uncover, and allow children to visualise, the structure of maths behind the numbers.

To use a rekenrek, children will slide beads from one side to the other. The beads always start on the right-hand side and, therefore, are always moved to the left. Children are taught that the beads are not moved one at a time, instead they take the 'one finger, one push' approach. This improves their number sense and moves them on from counting in ones.

Straws



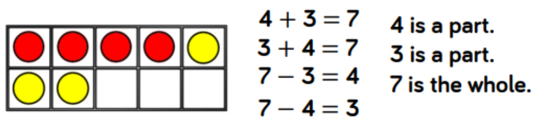
Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers. They are a good stepping stone to adding and subtracting with Base 10/Dienes.

Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

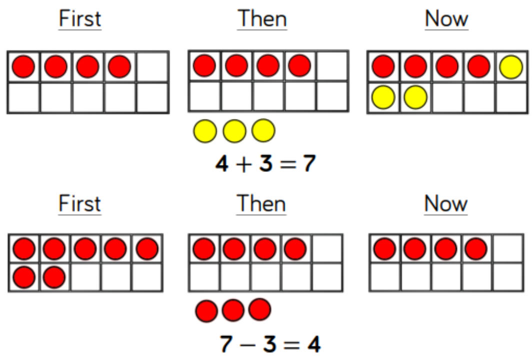
When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total. When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Ten Frame for additive reasoning (within 10)

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.



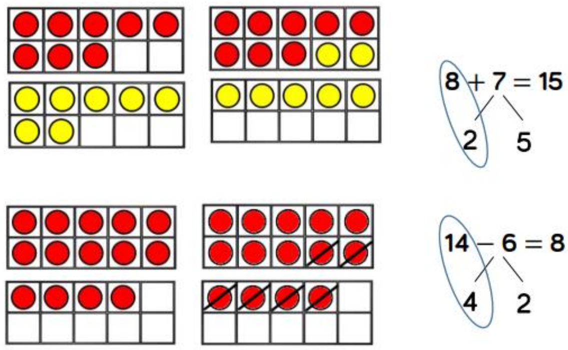
Using the language of parts and wholes represented by objects on the ten frame introduces children to **aggregation** and **partitioning**. Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.



Children can also use ten frames to look at **augmentation** (increasing a number) and **reduction** (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars

Ten Frame for additive reasoning (within 20)

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have **partitioned** one of the numbers to make 10, and makes links to effective mental methods of addition.



When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of **commutativity**.